

Space-Charge Simulations of the FNAL Main Injector

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performed in collaboration with

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Goals of the simulation effort

- Develop ability to model space-charge effects in MI over hundreds of thousands of turns
- Understand simulation parameter requirements (# of space-charge kicks/turn,...)
- Develop predictive capabilities that will be useful for Project-X design & design optimization studies
- Code benchmarking: compare results from IMPACT, MaryLie/IMPACT, and Synergia

Codes

- LBNL team using MaryLie/IMPACT and IMPACT
 - 3D parallel PIC codes
 - overlapping and complementary capabilities
 - IMPACT: highly optimized parallelization
 - ML/I: includes Lie algebraic map-based capabilities from MaryLie
- FNAL team using Synergia
 - For details see talk by E. Stern

Baseline Parameters

- Wiki page: <https://compacc.fnal.gov/projects/projects/show/mi-proj-x-sim>
- Lattice MAD8 format: mi20-egs.lat (slightly modified version of mi20-jfa.lat provided by Leo Michelotti)
- For the simulations that follow, we used these values:

Parameter	Main Injector	units
injection momentum $p_{\text{inj}}c$	8.9	GeV
x rms norm emittance $\varepsilon_{x,\text{rms},\text{norm}}$	7.85	mm-mrad
y rms norm emittance $\varepsilon_{y,\text{rms},\text{norm}}$	7.85	mm-mrad
t rms norm emittance $\varepsilon_{t,\text{rms},\text{norm}}$	0.0183	eV-sec
cavity rf voltage	55.6	kV in 18 cavities
cavity frequency	52.8124	MHz
protons/bunch N_p	1×10^{11}	
tunes Q_x, Q_y, Q_t	$26.425, 25.415, 9.195 \times 10^{-3}$	

Overview of simulation activities

- Comparison of zero-current lattice
- Studies of predictability/reversability with space charge
- Studies of required number of space-charge kicks per turn

Zero Current Calculation: no RF

```
#comments
  FNAL Main Injector -- mi20_mad-F_for_chef.lat 3/13/2008 with Eric Stern mods
    ryne/MainInjector/Tunes/NoRF
#menu
beam: beam, particle=proton, pc=8.9d0
units: units, type=static, l=1.0d0 ←
!
!
!
!   means scale the transverse momenta by the design momentum  $\gamma\beta m_p c$  (which is constant).
!   This choice of units is typical for systems without acceleration
mapout: ptm, matrix=3,poly=0
sanalyze: tasm !twiss analyze static map
fin: end, timers=true
!
! Half-cell lengths
!
LARCELL      = 17.288418
LSSCELL      = 17.288639
LDSCELL      = 12.966414
etc
etc
M_CAV: MARKER
etc
etc
RING_P_Q605: LINE=(SEC_6, SEC_1, SEC_2, SEC_3, SEC_4, SEC_5, SEC_6A)
#
#labor
RING_P_Q605
sanalyze
mapout
fin
```

Note: omission of "w=" means "use default scale angular frequency for the units chosen." For static units, default is $w=c/1$

scale length=1 meter. Hence, quantities involving particle coords x,y (transverse rms size, transverse rms emittance, etc.) are in meters. Also affects numeric value of map coefficients involving x,y.

! { mad8 description of lattice

Note: proton mass, $m_p=938.272013d6$

Results: Zero Current Calculation, no RF

twiss analysis of static map

horizontal tune = 0.425286542D+00
first order horizontal chromaticity = -4.92335652D+00
second order horizontal chromaticity = 1.72608686D+01

vertical tune = 0.415284670D+00
first order vertical chromaticity = -5.00401391D+00
second order vertical chromaticity = 1.12680905D+01

normalized anharmonicities
hhn= -130.0634996505588
vvn= -30.51057082874392
hvn= -276.6686293209266

matrix for map is :

-6.37526E-01	4.73175E+00	0.00000E+00	0.00000E+00	0.00000E+00	-1.14820E-01
-5.69175E-02	-1.14612E+00	0.00000E+00	0.00000E+00	0.00000E+00	2.30078E-03
0.00000E+00	0.00000E+00	-2.12865E+00	3.02708E+01	0.00000E+00	0.00000E+00
0.00000E+00	0.00000E+00	-6.15390E-02	4.05345E-01	0.00000E+00	0.00000E+00
8.00206E-03	1.20710E-01	0.00000E+00	0.00000E+00	1.00000E+00	2.96988E+01
0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00	1.00000E+00

Zero Current Calculation, no RF: Lattice Functions

```
#comments
FNAL Main Injector -- file mi20_mad-F_for_chef.lat 3/13/2008 with Eric Stern mods
ryne/MainInjector/LattFun/NoRF
#menu
beam: beam, particle=proton, pc=8.9d0
units: units, type=static, l=1.0d0
...
sanalyze: tasm, idata=2, ipmaps=0, isend=0, iwmaps=0
sq: sq, infile=20, logfile=0, istore=1, isend=1 !select quantities
!write selected quantities on fort.30
wsq: wsq, job=1, istore=1, ifile=30, lform=1, jform=2, isend=0
slice: autoslice, control=local
arc0: setarcrlen, s=0 !reset arc length to zero
autoconc: autoconcat, set=sandwich !autoconcat using the "sandwich" technique
autoapply: autoapply, name=analprint !automatically analyze & print after each slice
analprint, line=(sanalyze wsq)
fin: end, timers=true
...
```

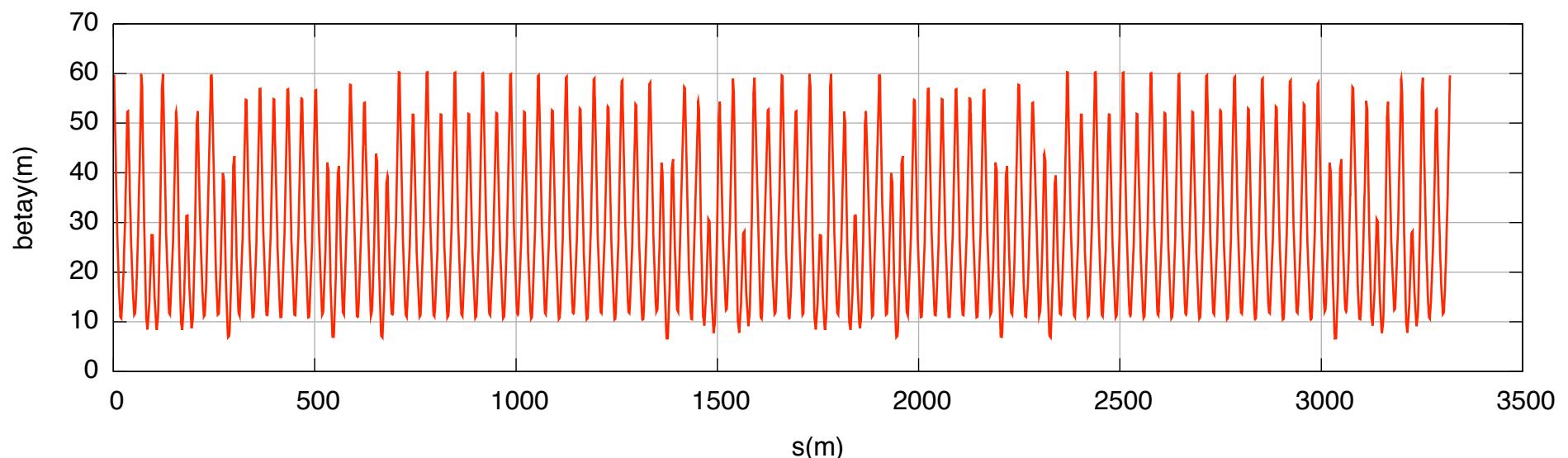
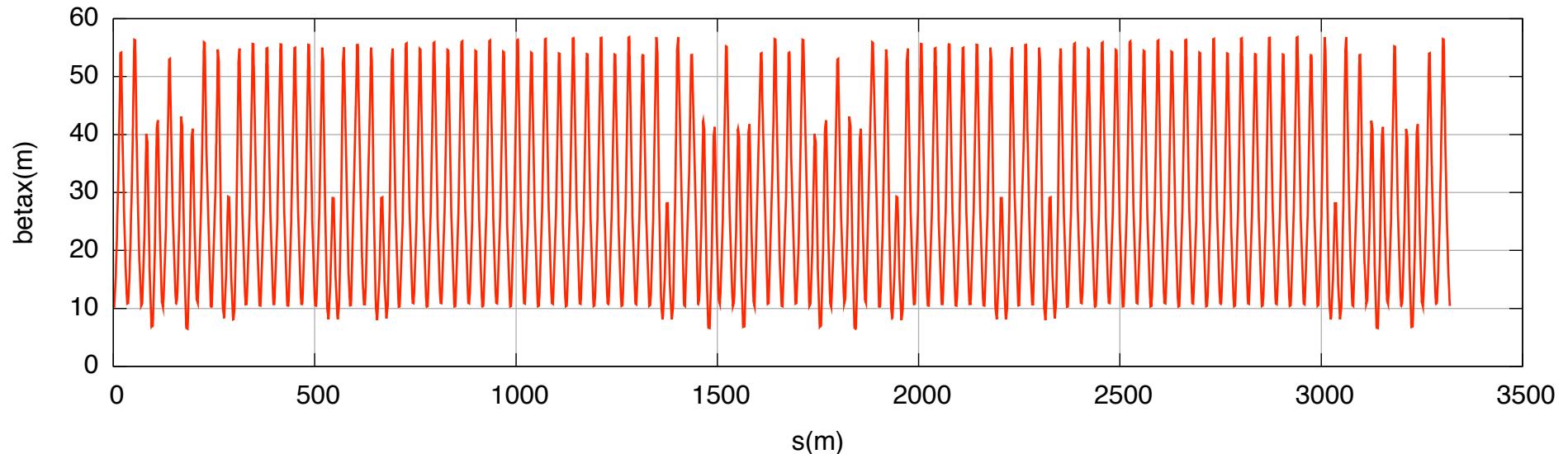


mad8 lattice

```
#labor
RING_P_Q605
sanalyze ! analyze the map
sq !selected quantities to be computed
arc0 !reset arc length to zero
wsq !write selected quantites (found by the previous tadm command)
autoconc !prepare to "sandwich" from now on
slice !prepare to slice elements from now on
autoapply
slice
RING_P_Q605
fin
```

contents of file fort.20:
1 bx
2 by
3 ax
4 ay
#

Zero Current Calculation, no RF: Lattice Functions



Zero Current Calculation: with RF

```
#comments
  FNAL Main Injector -- mi20_mad-F_for_chef.lat 3/13/2008 with Eric Stern mods
    ryne/MainInjector/Tunes/WithRF
#menu
beam: beam, particle=proton, pc=8.9d0
units: units, type=dynamic, l=1.0d0, w=2.d0*pi*52.8124d6
!
!
!
means scale the transverse momenta by  $m_p c$ .
This choice of units is typical for systems with acceleration. With this choice, emittances are automatically "normalized emittances"
!
mapout: ptm, matrix=3,poly=0
danalyze: tadm !twiss analyze dynamic map
fin: end, timers=true

! Half-cell lengths
!
LARCELL      = 17.288418
LSSCELL      = 17.288639
LDSCELL      = 12.966414
etc
etc
!M_CAV: MARKER
M_CAV: thinrfcav, volts=1.d6/18., freq=52.8124d6
etc
RING_P_Q605: LINE=(SEC_6, SEC_1, SEC_2, SEC_3, SEC_4, SEC_5, SEC_6A)
#
#labor
RING_P_Q605
danalyze
mapout
fin
```

scale angular frequency (in rad/sec) converts particle arrival time to phase.
Also affects numeric value of map coefficients involving t .

means scale the transverse momenta by $m_p c$.
This choice of units is typical for systems with acceleration. With this choice, emittances are automatically "normalized emittances"

mad8 description of lattice
with M_CAV replaced by
THINRFLCAV. This is a MaryLie
short rf cavity, but with the
option that, if the code is
tracking, it can track without
expanding $\sin(\omega t + \phi)$.
Default is to do map-based
tracking (Symplectic or Taylor)

Note: proton mass, $m_p = 938.272013d6$

Results: Zero Current Calculation with RF

twiss analysis of dynamic map

```
horizontal tune = 0.425286546D+00
vertical tune = 0.415284670D+00
temporal tune = 0.967397071D-02
```

normalized anharmonicities

```
hhn= -13.98510421153191
vvn= -3.498883445410293
ttn= -3.8191467679744434E-002
hvn= -29.44526529337716
htn= -1.1985779784337823E-004
vtn= -4.6401581627030685E-004
```

matrix for map is :

-6.37526E-01	4.98839E-01	0.00000E+00	0.00000E+00	6.45052E-06	-1.33982E-02
-5.39892E-01	-1.14612E+00	0.00000E+00	0.00000E+00	-1.22607E-06	2.54663E-03
0.00000E+00	0.00000E+00	-2.12865E+00	3.19126E+00	0.00000E+00	0.00000E+00
0.00000E+00	0.00000E+00	-5.83730E-01	4.05345E-01	0.00000E+00	0.00000E+00
8.85712E-03	1.40855E-02	0.00000E+00	0.00000E+00	9.98155E-01	3.83582E+00
-4.26424E-06	-6.78145E-06	0.00000E+00	0.00000E+00	-9.62007E-04	9.98152E-01

- Zero current results (tunes, chromaticities, linear lattice functions) agree between ML/I and Synergia

Zero Current Matching

- Using MaryLie's built-in normal form routines it is straightforward to generate a zero-current matched beam for a periodic transport system:
 - Normalize 1-turn map: $M = A^{-1}NA$ (A = normalizing map; N =normal form)
 - Let $\zeta = (x, p_x, y, p_y, t, p_t)$ and consider a function $g_{\text{tori}}(\zeta)$ that depends only on $I_x = (x^2 + p_x^2), I_y = (y^2 + p_y^2), I_t = (t^2 + p_t^2)$, i.e. it depends only on radii in each 2d phase plane
 - Then $f(\zeta) = g_{\text{tori}}(A \zeta)$ is a matched beam.
 - Proof: Let $f(\zeta) = g_{\text{tori}}(A \zeta)$. Then distribution after one turn is given by
$$f(M^{-1}\zeta) = g_{\text{tori}}(AN A^{-1} \cdot A(x^2 + p_x^2), (y^2 + p_y^2), (t^2 + p_t^2)) =$$
$$g_{\text{tori}}(AN(x^2 + p_x^2), (y^2 + p_y^2), (t^2 + p_t^2)) =$$
$$g_{\text{tori}}(A(x^2 + p_x^2), (y^2 + p_y^2), (t^2 + p_t^2))$$
- We generated a numerical distribution using this approach, performed element-by-element tracking with ML/I, verified match, placed distribution at NERSC for use by collaborators

Finding a zero current matched beam

(linear matching and linear tracking)

```

#comments
FNAL Main Injector -- mi20_mad-F_for_chef.lat 3/13/2008 with Eric Stern mods
ryne/ZeroCurrLinearMatchAndTrack
#menu
beam: beam, particle=proton, pc=8.9d0
units: units, type=dynamic, l=1.0d0, w=2.d0*pi*52.8124d6
!
gengauss6d: bgen, dist=6 & norm 95% emittance in x,y =  $15 \times 10^{-6} \pi$  meter-rad. Factor  $\pi/6$  converts to norm rms emittance in meter-rad
    maxray=1000000, xemit=15.d-6*(pi/6.) yemit=15.d-6*(pi/6.) temit=0.036667*(1./6.)/p0sc tptmax=3.
!
!
!
norm 95% emittance in longitudinal phase space = 0.036667 eV-sec.
Factor 1/6 converts to norm rms emittance. Factor 1/p0sc converts to ML/l dimensionless units
!
!
!
dump:particledump,min=1,max=100000,file=dump,precision=9,close=true,nunits=0,sequencelength=1000
prntmoms: moments, precision=9, nunits=1
prntmax: maxsize, precision=9, nunits=1
prntall: line=(prntmoms, prntmax)
track: autotrack, type=taylor1

xprof: profile1d, column=1, file=xprofile, sequencelength=1000, nunits=1, rwall=.02
pxprof: profile1d, column=2, file=pxprofile, sequencelength=1000, nunits=1, rwall=.015
yprof: profile1d, column=3, file=yprofile, sequencelength=1000, nunits=1, rwall=.04
pyprof: profile1d, column=4, file=pyprofile, sequencelength=1000, nunits=1, rwall=.015
tprof: profile1d, column=5, file=tprofile, sequencelength=1000, nunits=1, rwall=pi
ptprof: profile1d, column=6, file=ptprofile, sequencelength=1000, nunits=1, rwall=.05

profiles: line=(xprof, pxprof, yprof, pyprof, tprof, ptprof)
applymap: raytrace, type=taylor1, ntrace=1, nwrite=0
sdnor: dnor, isend=0 ! "silent" dnor command
getscripta: gbuf, nbuf=1, replace=true
clear: iden
setref0: setreftraj, restart=true
fin: end, timers=true
...
...} mad8 lattice
...} }
```

#labor
RING_P_Q605
sdnor
getscripta
gengauss6d
applymap
dump
profiles
clear
setref0
prntall
!verify match by tracking
track
RING_P_Q605
dump
profiles
prntall
fin

norm 95% emittance in x,y = $15 \times 10^{-6} \pi$ meter-rad. Factor $\pi/6$ converts to norm rms emittance in meter-rad

norm 95% emittance in longitudinal phase space = 0.036667 eV-sec.

Factor 1/6 converts to norm rms emittance. Factor 1/p0sc converts to ML/l dimensionless units

cut off longitudinal distribution at 3σ radius in 2D longitudinal phase space

code will produce a linearly matched distribution of particles

Results: Finding a zero current match using a linear matching procedure and linear tracking

First line of each file shows the initial matched value; second line is after 1 turn:

xrms.out:					
s	$\sqrt{\langle x^2 \rangle}$	$\sqrt{\langle p_x^2 \rangle}$	$\langle x p_x \rangle$	ε_x	
0.00000000E+00	2.94464125E-03	9.58063544E-03	-4.90500624E-01	2.45846832E-05	
3.31941869E+03	2.94435653E-03	9.57817140E-03	-4.90015872E-01	2.45836735E-05	
xmax.out:					
s	x_{\min}	x_{\max}	$p_{x,\min}$	$p_{x,\max}$	
0.00000000E+00	-1.40641959E-02	1.45410710E-02	-4.79405928E-02	4.37295330E-02	
3.31941869E+03	-1.40097229E-02	1.39206544E-02	-4.38399086E-02	4.64032123E-02	
yrms.out:					
s	$\sqrt{\langle y^2 \rangle}$	$\sqrt{\langle p_y^2 \rangle}$	$\langle y p_y \rangle$	ε_y	
0.00000000E+00	7.02462484E-03	9.40698980E-03	9.28251101E-01	2.45789442E-05	
3.31941869E+03	7.02297470E-03	9.40154457E-03	9.28129680E-01	2.45789442E-05	
ymax.out:					
s	y_{\min}	y_{\max}	$p_{y,\min}$	$p_{y,\max}$	
0.00000000E+00	-3.21736724E-02	3.35761048E-02	-4.68342495E-02	4.34139095E-02	
3.31941869E+03	-3.57998149E-02	3.35779056E-02	-4.61366591E-02	4.37090053E-02	
trms.out:					
s	$\sqrt{\langle t^2 \rangle}$	$\sqrt{\langle p_t^2 \rangle}$	$\langle t p_t \rangle$	ε_t	
0.00000000E+00	1.78537621E-09	9.74506612E+06	-1.01644839E-04	1.73986092E-02	
3.31941869E+03	1.78536872E-09	9.74510737E+06	-8.59858903E-05	1.73986098E-02	
tmax.out:					
s	t_{\min}	t_{\max}	$p_{t,\min}$	$p_{t,\max}$	
0.00000000E+00	-5.49529254E-09	5.49540382E-09	-2.99884232E+07	2.99602868E+07	
3.31941869E+03	-5.49338586E-09	5.49335763E-09	-2.99256574E+07	2.99832099E+07	

Comments

- Zero-current match is excellent, as expected
- In reality the longitudinal beam extent is large, so linear tracking (i.e. keeping first order in time-of-flight terms) is a poor approx
- ML/I has a “thinrfcav” element allows nonlinear tracking with no expansion of $\sin(\omega t)$
 - M_CAV: thinrfcav, volts=1.d6/18., freq=52.8124d6, fullynonlinear=true
- For these studies, transverse nonlinearities (quad hard-edge fringe fields, dipole fringe fields) are turned off.
- Hence, for these studies, it is sufficient to use linear tracking for drifts, quads, and dipoles, and to use the fully nonlinear thinrfcav option for the rf cavities.

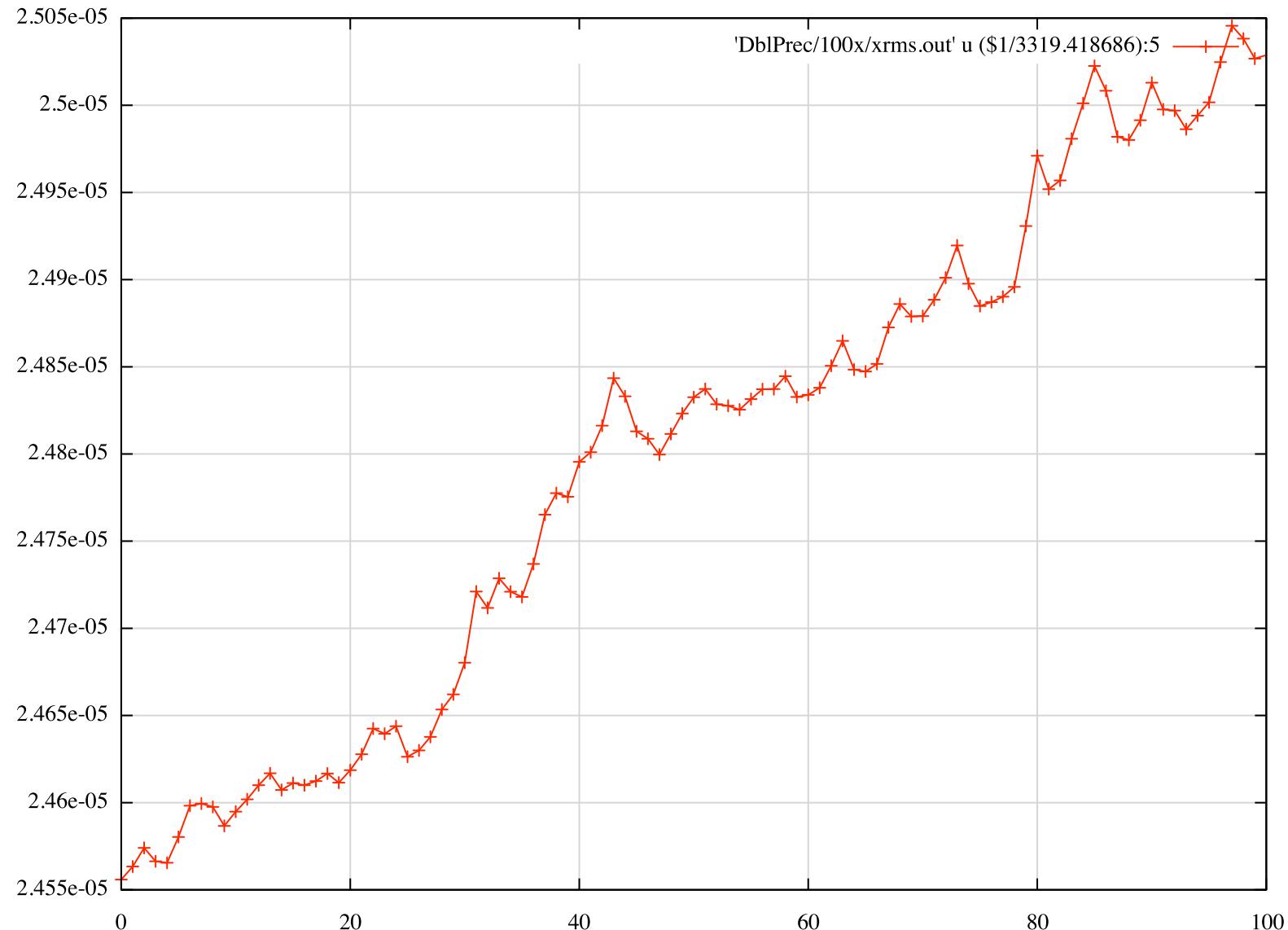
Simulation Challenges

- We are interested in predicting behavior over a few hundred thousand turns
- The space-charge driven emittance evolution per turn is very small, <.01%
- Two key issues:
 - what is our confidence in predicting the cumulative effect of weak space-charge forces acting for very long times?
 - how many space-charge kicks/turn do we need for the results to converge?
- The first issue is more subtle than it might appear at first glance:
 - PIC codes normally have a certain amount of numerical noise/statistical fluctuations due to use of finite # of macroparticles. For moderate space charge (tune depression of a few 10's of percent), the effect of the noise is small compared to what we are trying to model, so it is not a problem; but, we would not normally trust the results to better than of order a percent, which is fine in that regime. In *this* case the space charge is very weak, the effect per turn might be the same order as (or smaller than) the numerical noise; what we really care about is long-term impact of the physical (not numerical) effect.
 - To have confidence in our predictions requires careful testing. One such test involves reversability tests to look for physical or numerical chaos

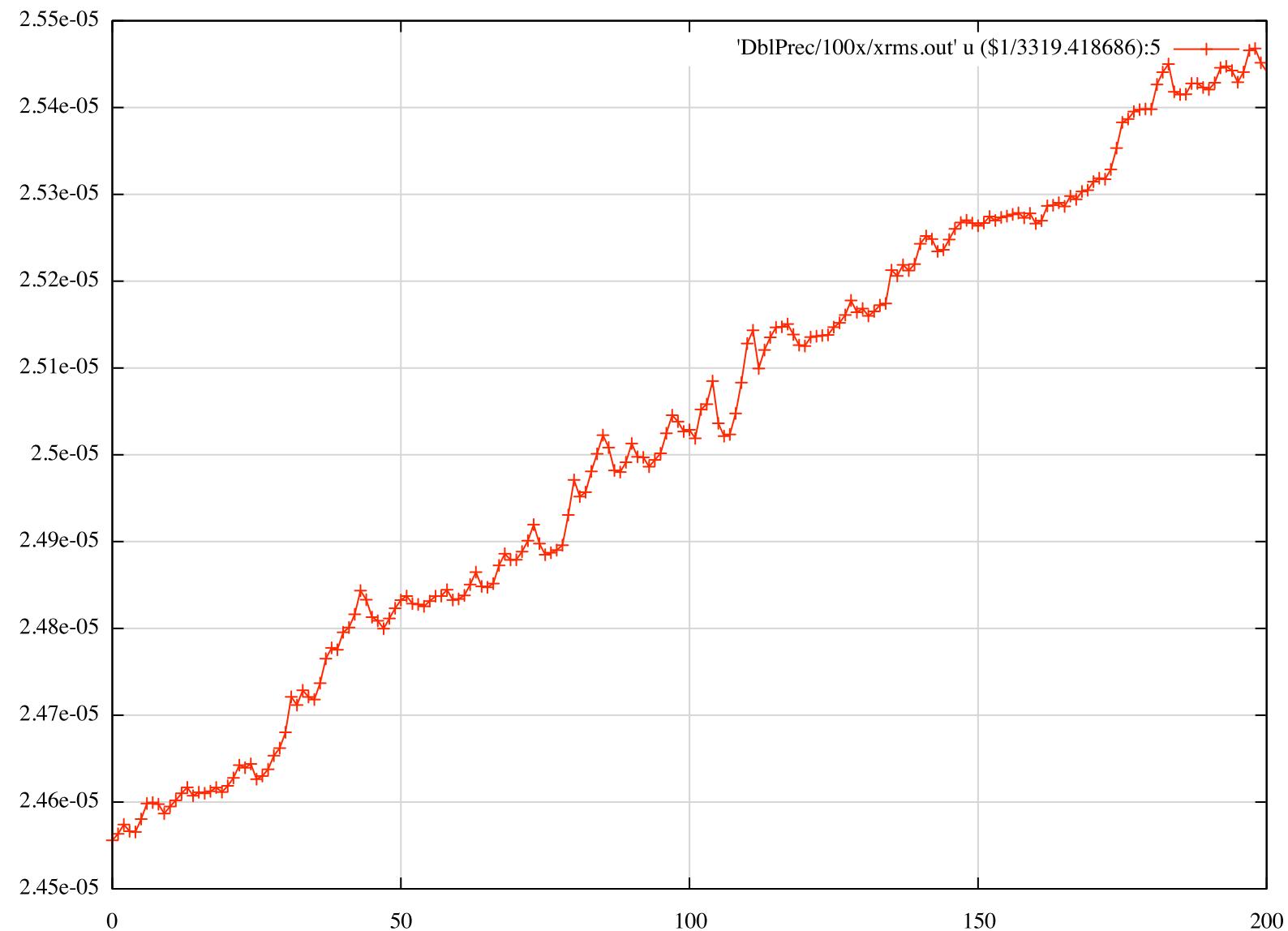
Reversability tests

- For non-chaotic systems, the PIC codes we are using are formally reversible, even in the presence of space charge
 - to reverse, first perform a forward simulation, then set $px \rightarrow -px$, $py \rightarrow -py$, $t \rightarrow -t$, then perform backward simulation to the beginning
- lack of reversability would imply chaos in the numerical dynamical system (assuming additive roundoff is not an issue; chaotic systems amplify roundoff exponentially)
- Our early simulations showed a lack of reversability, which was surprising given the very weak space charge
- We discovered that the Poisson solver we were using was sensitive to roundoff when the beam had a high aspect ratio
 - in this case, the beam is $\sim 50\text{-}100x$ longer than its radius
- Problem solved by using quad precision for the Integrated Green function (IGF)Poisson solver

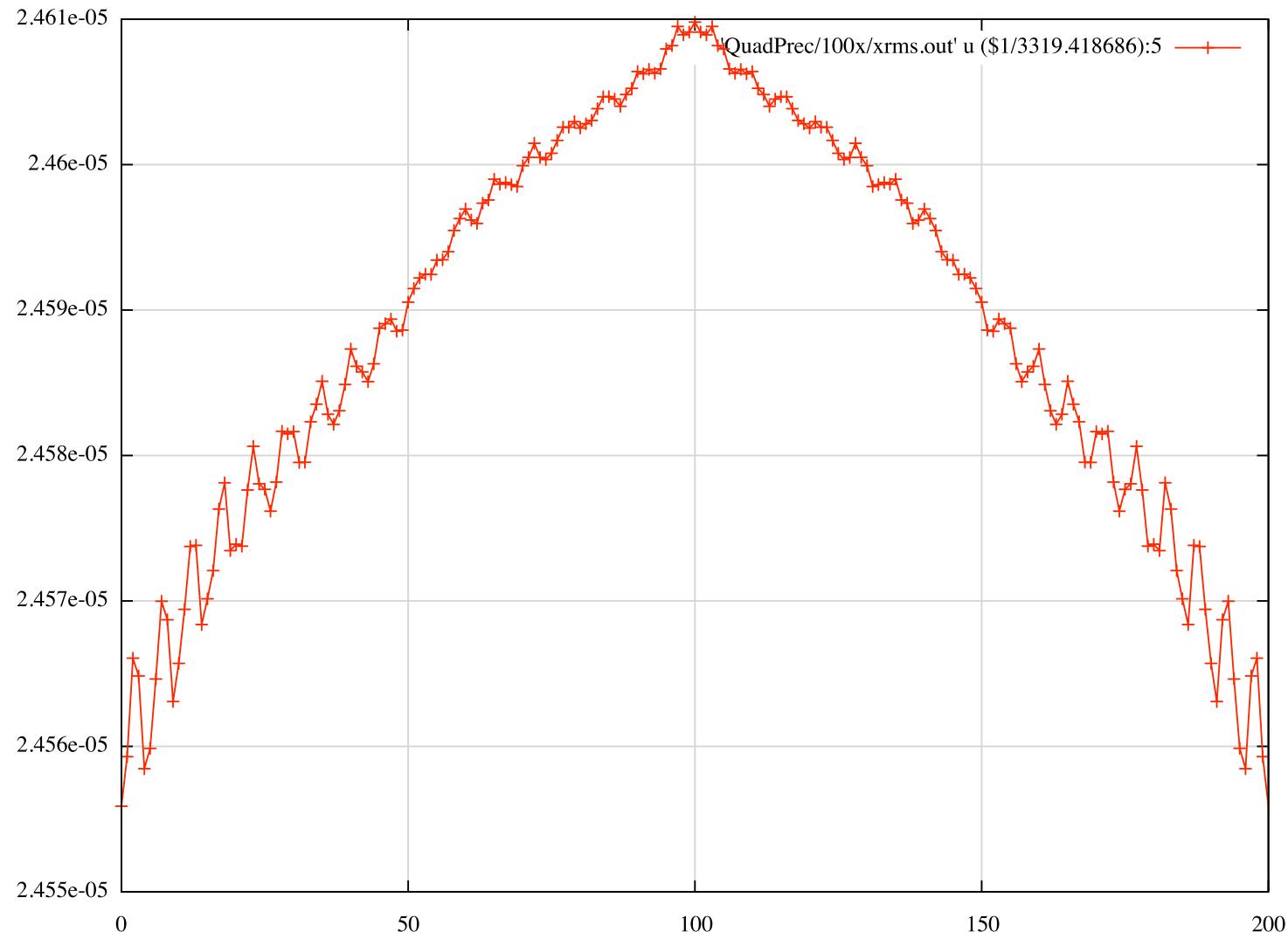
Early simulation of emittance growth over 100 turns using 30 space-charge kicks/turn [~100m slices]. Should this result be trusted???



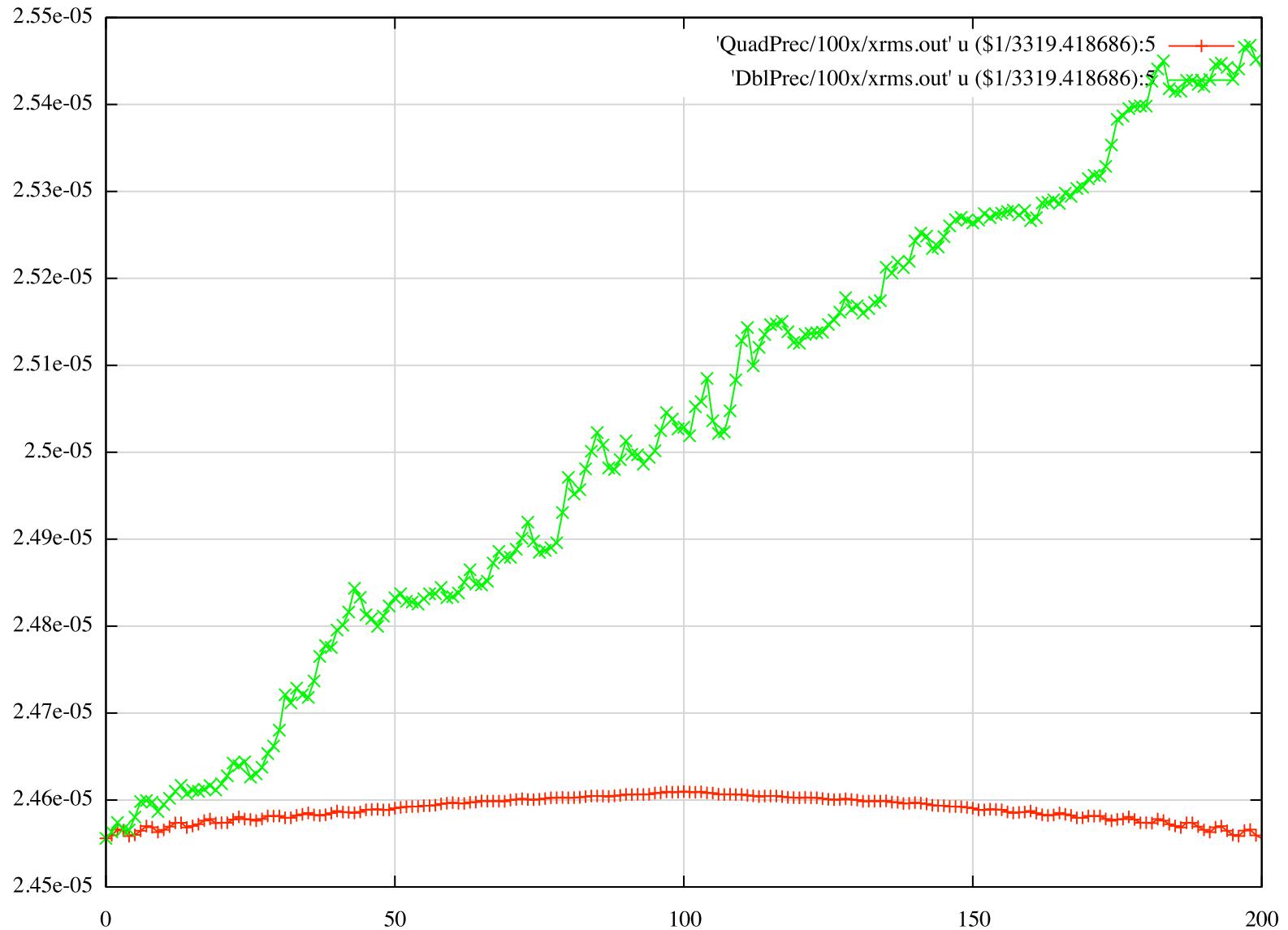
Here is what happens if the simulation is reversed after 100m. Note a total lack of reversability.



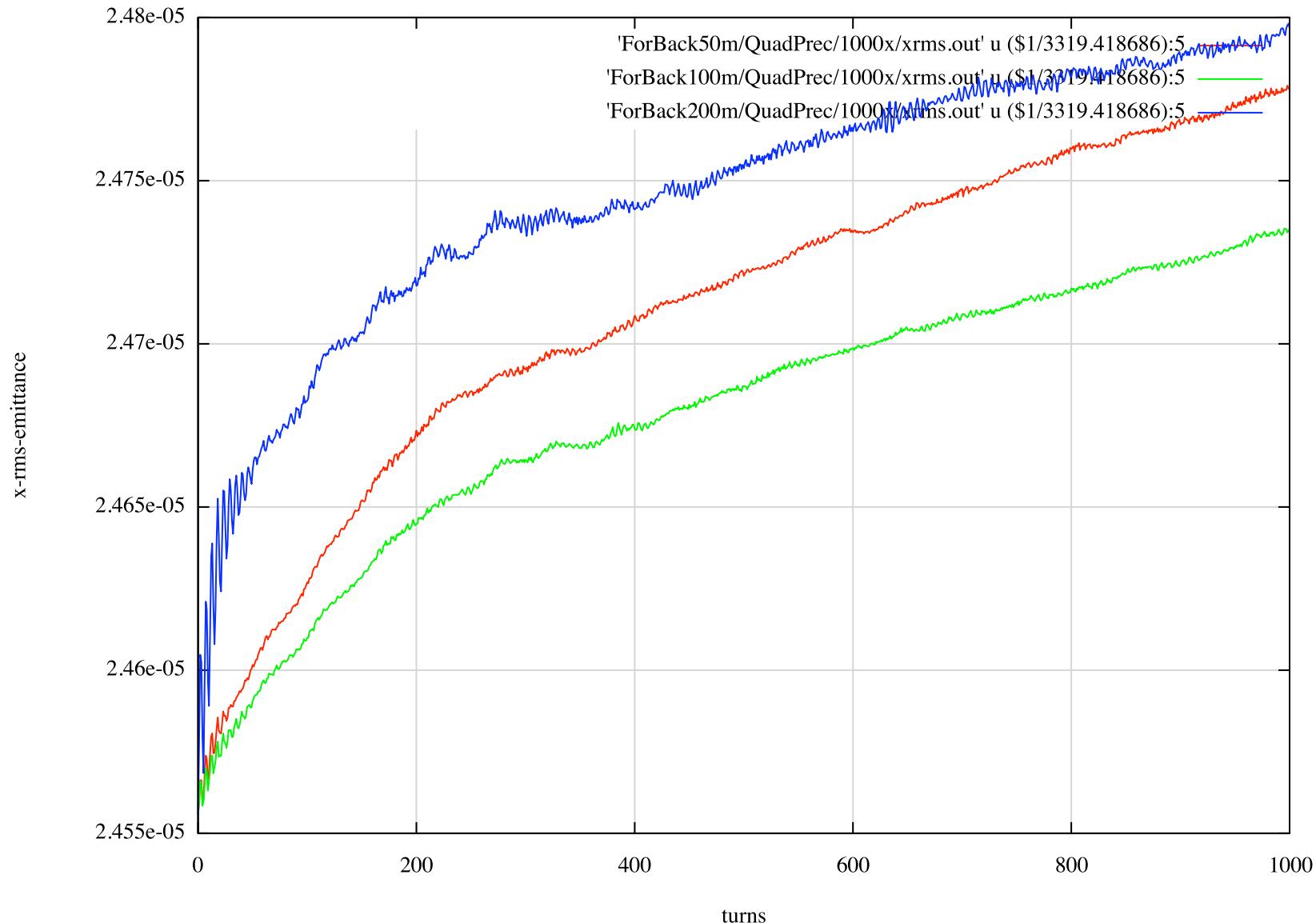
Here is the identical simulation but, instead of using double precision for the entire calculation, this was done with a quad precision version of the integrated Green function routines. This simulation is reversible. The previous result was garbage.



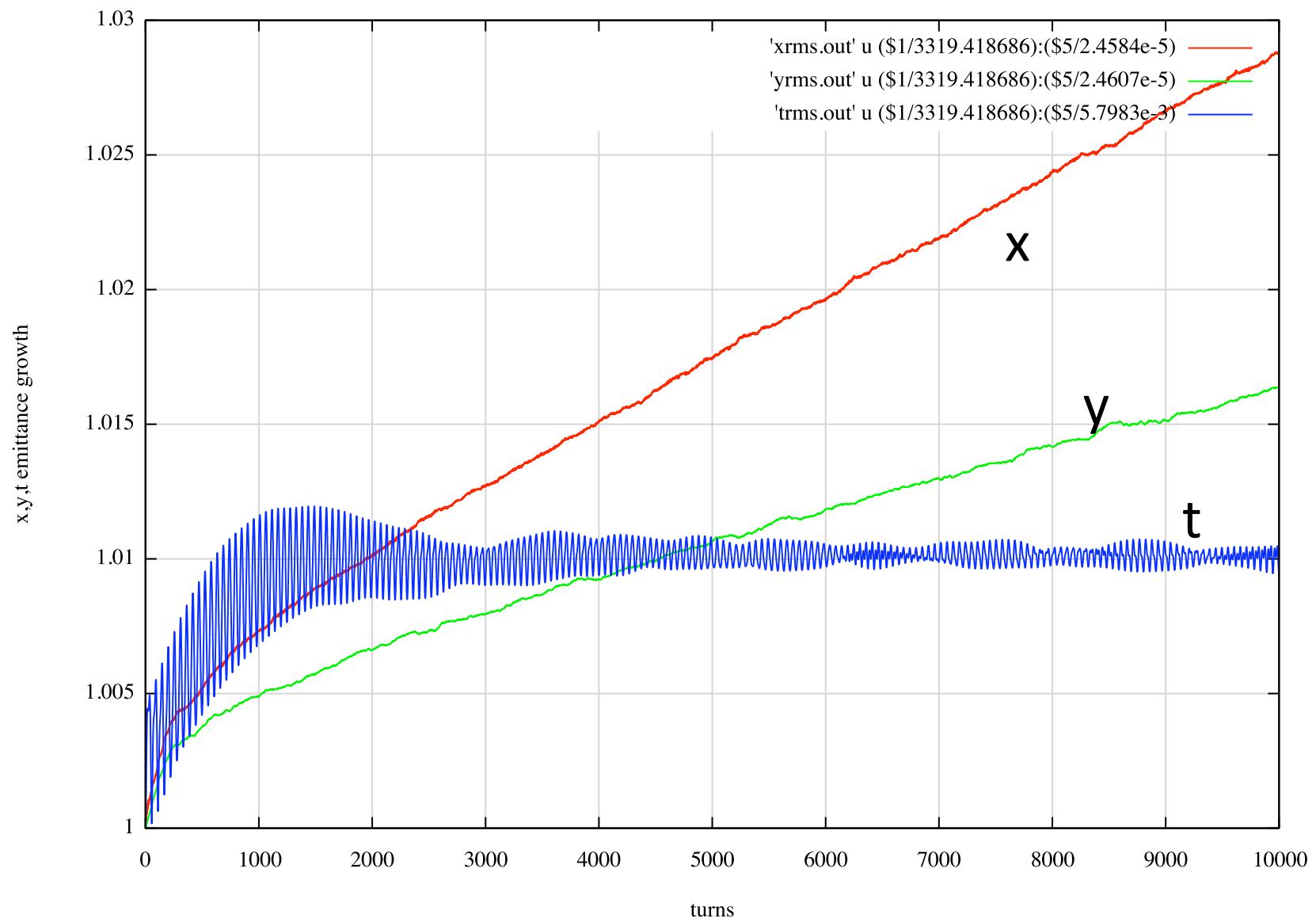
Previous results on the same plot. Double vs. Quad precision IGF



Comparison of emittance growth over 1000 turns (with quad precision IGF) using 15 kicks/turn [\sim 200m slices], 30 kicks/turn [\sim 100m slices], and 60 kicks/turn [\sim 50 m slices]



Horizontal, vertical, and longitudinal (x-,y-,t-) emittance growth (with reduced bunch length) over 10,000 turns using 30 kicks/turn [$\sim 100\text{m}$ slices]



- Our results show “reasonable” behavior for 15, 30, and 60 kicks/turn.
- But convergence study of E. Stern (see later talk) indicates numerical convergence requires of order 700 kicks/turn.
- Required # of kicks/turn is still an open question, since it depends on strongly on what we are trying to predict
 - want to predict quantitative emittance growth to high accuracy? predict stability? predict loss rate?
- For now we are being conservative and using 728 kicks/turn

Space-charge studies using 728 kicks/turn

full bunch length used

524288 particles

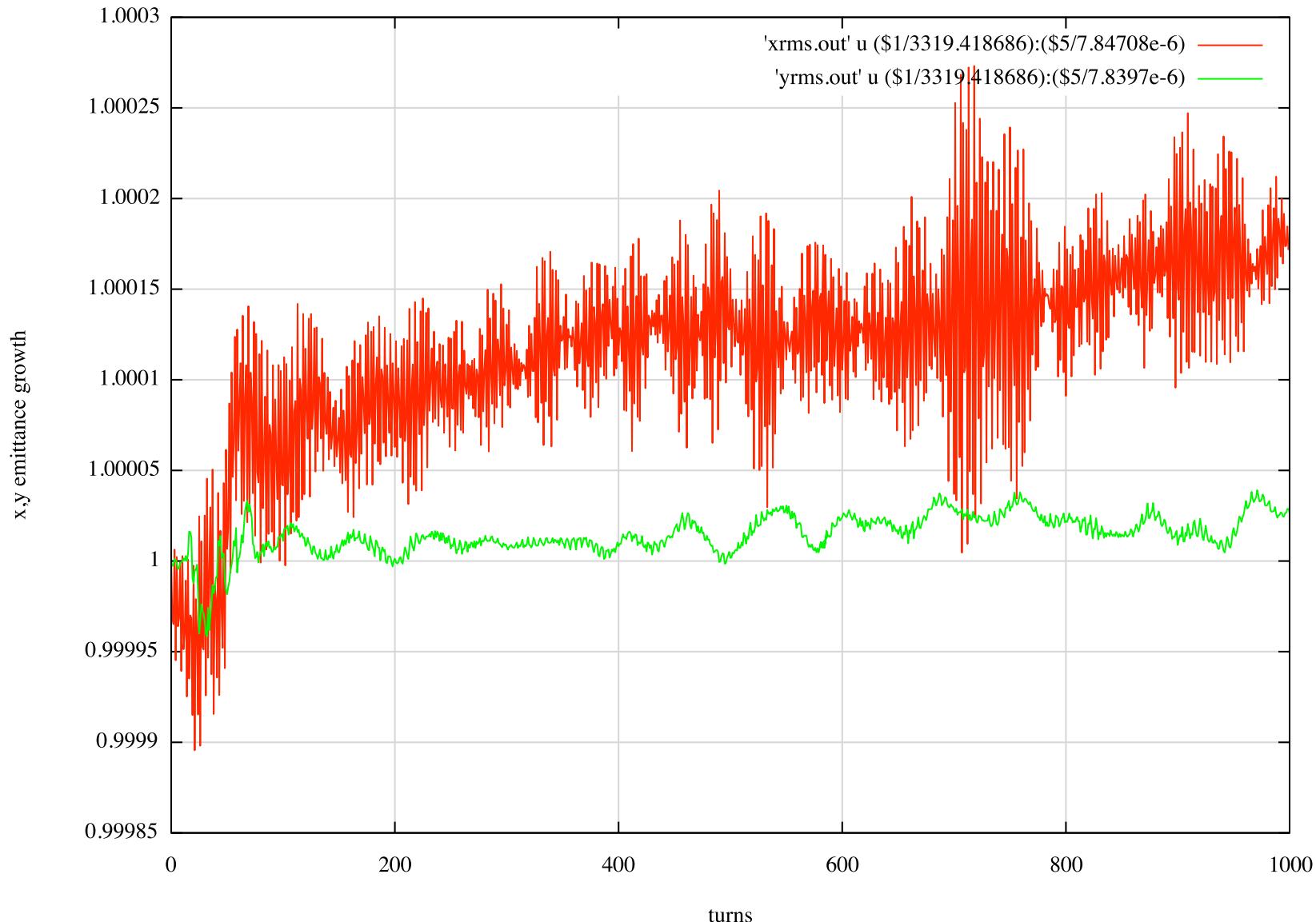
32x32x32 grid and 32x32x128 grid

includes particle losses

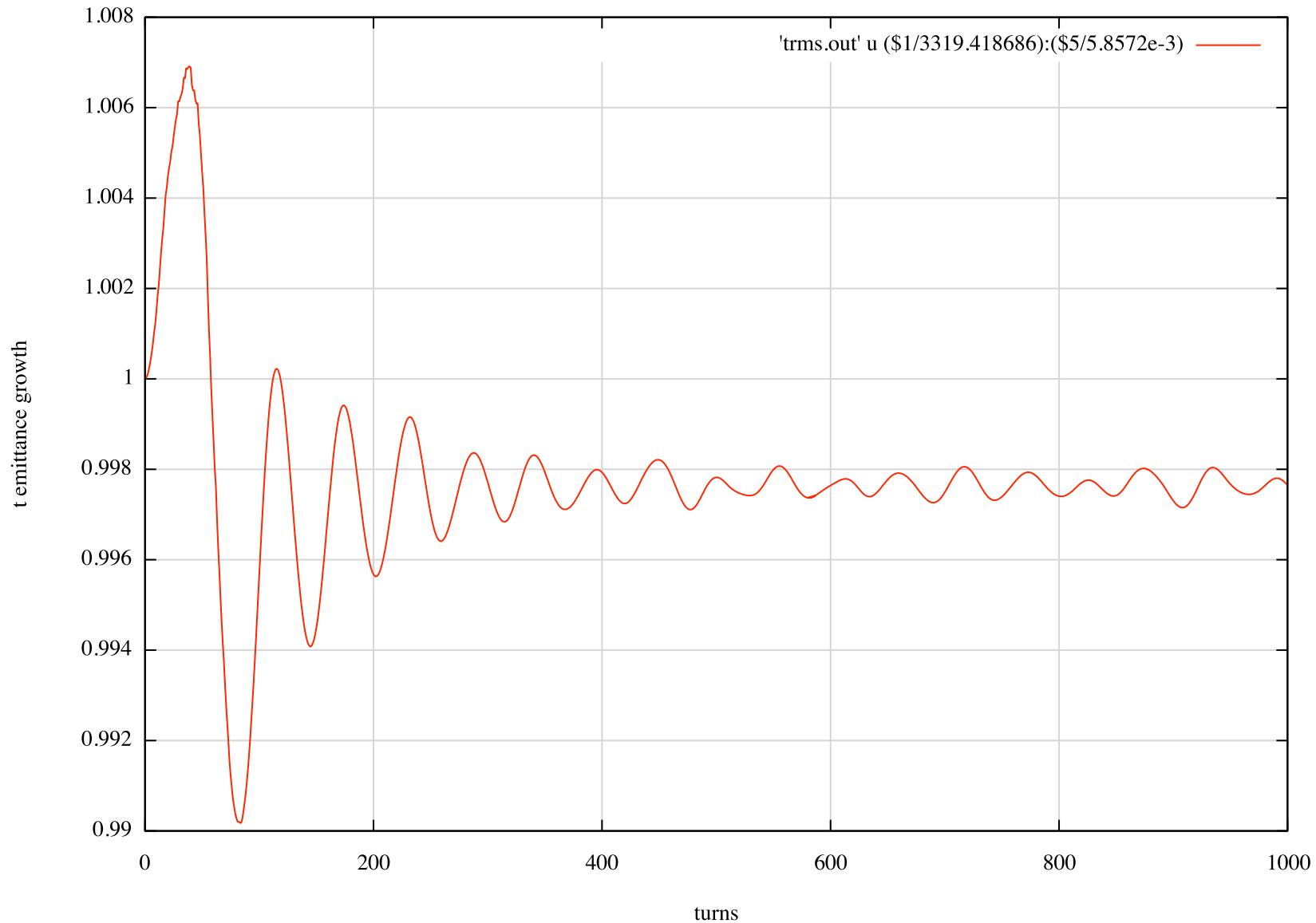
Tracking with space charge

```
#comments
FNAL Main Injector -- mi20_mad-F_for_chef.lat 3/13/2008 with Eric Stern mods
ryne/ZeroCurrMatchAndSCTrack
#menu
beam: beam, particle=proton, pc=8.9d0, bfreq=52.8124d6, bcurr=0.846d0 ← charge per bunch=bcurr/bfreq
units: units, type=dynamic, l=1.0d0, w=2.d0*pi*52.8124d6
!
initpoisson: poissonfftw, nx=32,ny=32,nz=32, densityfunction=linear
!
...
...
...
...
<same as before>
...
...
...
...
...
...
initpoisson: poisson, nx=32, ny=32, nz=32
RING_P_Q605prnt: line=(RING_P_Q605 prntall)
#labor
<same as before, generate zero current matched beam>
!turn on space charge
initpoisson
!track element-by-element through the lattice:
track
1000*RING_P_Q605prnt ← track through 100 turns, print diagnostic quantities
dump
profiles
fin
```

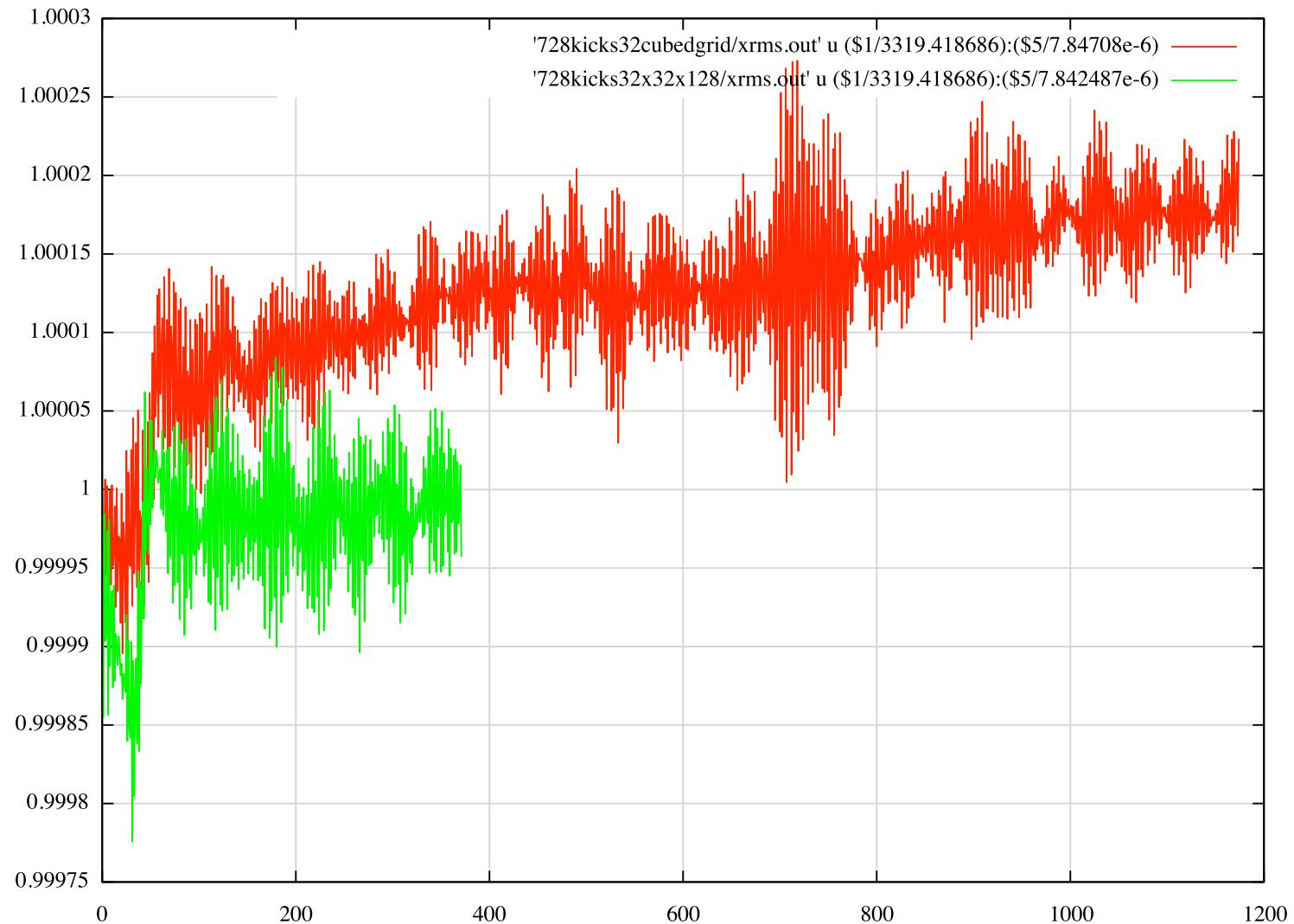
Tracking with space charge: Emittance Growth
32x32x32 grid, 728 space-charge kicks/turn
x- and y-emittance growth



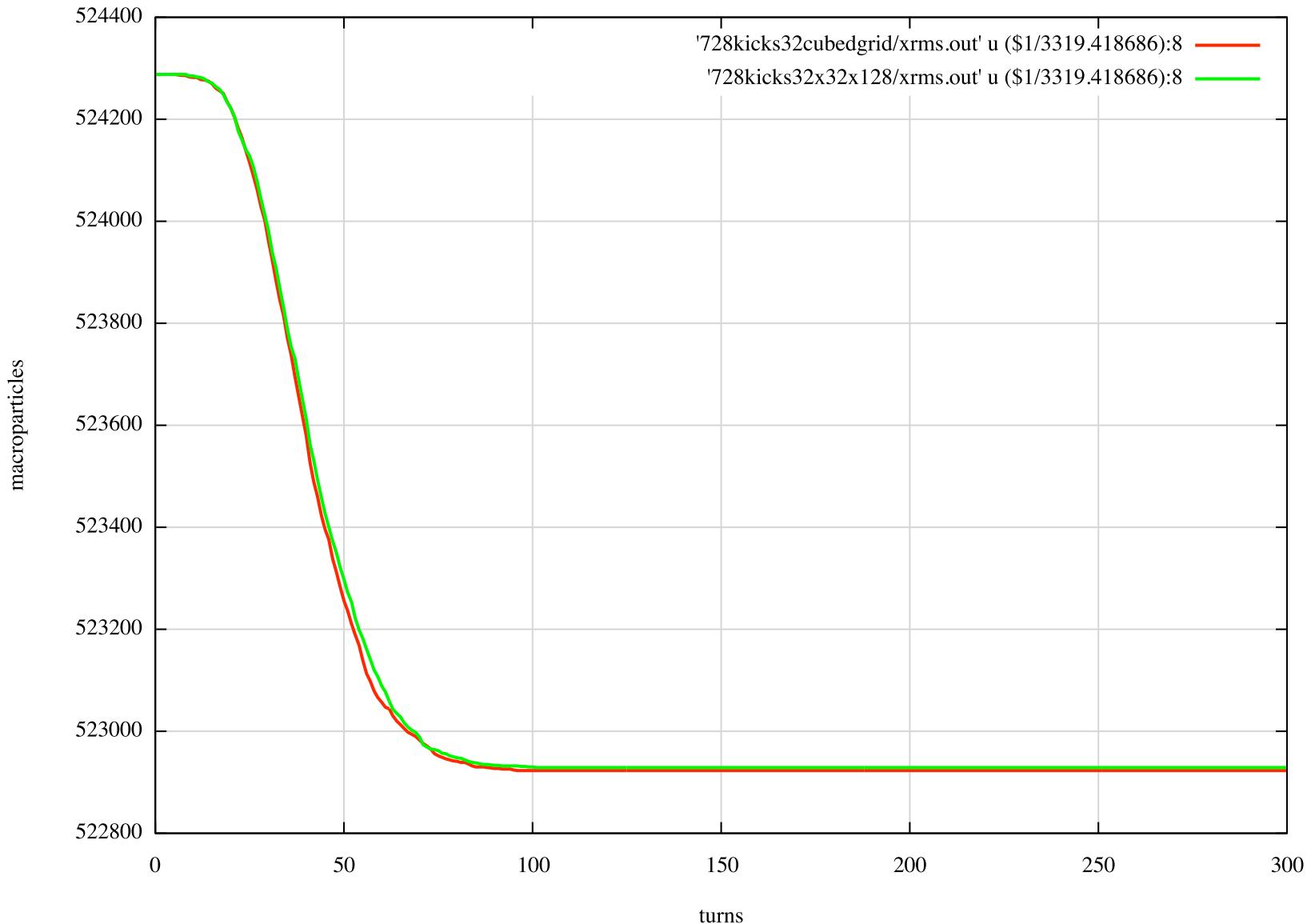
Tracking with space charge: Emittance Growth
32x32x32 grid, 728 space-charge kicks/turn
t-emittance growth



Tracking with space charge:
32x32x32 grid vs 32x32x128, 728 space-charge kicks/turn
x-emittance growth



Tracking with space charge:
32x32x32 grid vs 32x32x128, 728 space-charge kicks/turn
particle loss



Next Steps

- Possible next steps, depending on feedback from FNAL:
 - tune footprint
 - switch to using conducting-pipe Poisson solver
 - complete study of required # of space-charge kicks/turn
 - assure convergence with respect to other parameters (# macroparticles, s-c grid,...)
 - performance optimization to reach ~100K turns
 - more realistic physics: nonlinear rf cavity modeling, magnet fringe fields,...
 - model ramping
 - model H- injection
 - ... others?